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Deuteron Magnetic and Quadrupole Moments with a Poincaré Covariant Current Operator in the Front-Form Dynamics

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Abstract

The deuteron magnetic and quadrupole moments are unambiguously determined within the front-form Hamiltonian dynamics, by using a new current operator which fulfills Poincaré, parity and time reversal covariance, together with hermiticity and the continuity equation. For both quantities the usual disagreement between theoretical and experimental results is largely removed.

The deuteron is a good system for a test of relativistic approaches devoted to the investigation of hadron electromagnetic (em) properties (see, e.g., [1, 2, 3, 4, 5] and Refs. quoted therein) and in particular of the accuracy of the one-body impulse approximation (IA) for the current operator. It is usually believed that effects beyond IA, e.g. meson-exchange currents, $N\bar{N}$ -pair creation terms (Z graphs), and isobar configurations in the deuteron wave function are important for the explanation of existing data. However, these effects are essentially model dependent [6] and, furthermore, obviously depend on the reference frame (see, e.g., Refs. [7, 8]).

A widely adopted framework for relativistic investigations of deuteron em properties is the front-form Hamiltonian dynamics (FFHD) [9, 10], where only the two-nucleon state is considered and the wavefunction of the system factorizes, for any front-form boost, in an eigenfunction of the total momentum times an intrinsic wavefunction. Thus, deuteron em form factors, determined from three independent matrix elements of the current, are given in terms of elastic em nucleon form factors and deuteron internal wave function. In the FFHD, the one-body approximation was usually applied only to the relevant matrix elements of the *plus* component of the current in the reference frame where $q^+ = 0$ (q is the momentum transfer), while the other ones were properly defined in order to fulfill all the required properties (see, e.g., [1, 7, 11]). In this letter, we will consider the Breit reference frame where the three-momentum transfer is along the spin quantization axis, which allows one to exploit the symmetry of the problem and to calculate all the non-vanishing matrix elements of the current by the same rules.

Following Ref.[8], let us consider the current which in the Breit frame where $\vec{P}_\perp = \vec{q}_\perp = 0$ has the form

$$\begin{aligned} j^\mu(K\vec{e}_z) &= \frac{1}{2} \{ \mathcal{J}^\mu(K\vec{e}_z) + L_\nu^\mu[r_x(-\pi)] \exp(i\pi S_x) \mathcal{J}^\nu(K\vec{e}_z)^* \exp(-i\pi S_x) \} \\ \mathcal{J}^+(K\vec{e}_z) &= \mathcal{J}^-(K\vec{e}_z) = \langle \vec{P}_\perp = 0, P'^+ | \Pi J_{free}^+(0) \Pi | \vec{P}_\perp = 0, P^+ \rangle \\ \vec{\mathcal{J}}_\perp(K\vec{e}_z) &= \langle \vec{P}_\perp = 0, P'^+ | \Pi \vec{J}_{free\perp}(0) \Pi | \vec{P}_\perp = 0, P^+ \rangle. \end{aligned} \quad (1)$$

In Eq. (1), Π is the projector onto the subspace of deuteron bound states $|\chi_1\rangle$ of mass m_d and spin 1, $J_{free}^\mu(0) = J_p^\mu(0) + J_n^\mu(0)$ is the one-body current, $|\vec{P}_\perp, P^+\rangle$ is an eigenstate of the total deuteron momentum, $\vec{P}_\perp \equiv (P_x, P_y)$, $P^+ \equiv (P_0 + P_z)/\sqrt{2} = \frac{1}{\sqrt{2}}[(m_d^2 + K^2)^{1/2} - K]$, $P'^+ = \frac{1}{\sqrt{2}}[(m_d^2 + K^2)^{1/2} + K]$, $K = Q/2$, $Q^2 = -q_\mu^2$ and $q = P' - P$; $L[r_x(-\pi)]$ is the element of the Lorentz group corresponding to a rotation of $-\pi$ around the x axis, S_x is the x component of the front-form spin operator, and $*$ means Hermitian conjugation in internal space. From Eq. (1), one can obtain the expression of the current in any other reference frame by applying the proper transformations (see, e.g., [8]). This current operator fulfills extended Poincaré covariance, hermiticity and the *charge normalization*, as well as current conservation [8]. The second term in the first line of Eq. (1), which ensures hermiticity, introduces two-body terms in the current, because of the presence of S_x (see below).

A relevant result of our approach is that the extraction of elastic em form factors is no more plagued by the ambiguities, related to the so called "angular condition", which are present when the free current is used in the $q^+ = 0$ frame (see, e.g., [11, 7, 12]). In this case, one has four independent matrix elements of the current, while the em form factors are three [11]. Differently, in our model (Eq. (1)), it turns out [8] that only *three matrix elements* $j_{S'_z, S_z}^\mu = \langle m_d 1 S'_z | j^\mu(K\vec{e}_z) | m_d 1 S_z \rangle$ are independent (e.g., $j_{0,0}^+$, $j_{1,1}^+$, and $j_{1,0}^x$). Therefore, there is no longer any freedom in the construction of the three em form factors. These matrix elements (as well as any other one) can be easily obtained by Eq. (1) in terms of the matrix elements $\mathcal{J}_{S'_z, S_z}^\mu = \langle m_d 1 S'_z | \mathcal{J}^\mu(K\vec{e}_z) | m_d 1 S_z \rangle$. Indeed,

by using the properties of the Wigner D-functions, one can show that the two terms in the first line of Eq. (1) are equal for the *plus* component ($j_{0,0(1,1)}^+ = \mathcal{J}_{0,0(1,1)}^+$), while for the x component they yield $\mathcal{J}_{1,0}^x/2$ and $-\mathcal{J}_{0,1}^x/2$, respectively ($j_{1,0}^x = [\mathcal{J}_{1,0}^x - \mathcal{J}_{0,1}^x]/2$) [8].

As a test of our current, we evaluate the deuteron form factors at $Q^2 = 0$, namely the magnetic moment, μ_d , and the quadrupole moment, Q_d , which are not affected by the uncertainties in the knowledge of the neutron em form factors at finite momentum transfers. The deuteron moments are a longstanding problem in nuclear physics, since it was not possible to reconcile in a coherent approach theoretical and experimental values for both quantities at the same time, by changing the tensor content of the nucleon-nucleon ($N - N$) interaction, or considering two-body current contributions, both in non-relativistic and in relativistic frameworks [2, 13, 14, 5]. Our preliminary results for the deuteron form factors at $Q^2 \neq 0$ can be found in [15].

By using the properties [8] of the matrix elements of $j_{S'_z, S_z}^\mu$ the deuteron form factors can be written in terms of the matrix elements $\mathcal{J}_{S'_z, S_z}^\mu$ [15]. Then, the magnetic moment, in nuclear magnetons, is given by

$$\mu_d = \frac{(m_p \sqrt{2})}{m_d} \lim_{Q \rightarrow 0} \frac{1}{Q} \frac{[\mathcal{J}_{1,0}^x - \mathcal{J}_{0,1}^x]}{2}, \quad (2)$$

while the quadrupole moment is

$$Q_d = \frac{\sqrt{2}}{m_d} \lim_{Q \rightarrow 0} \frac{1}{Q^2} [\mathcal{J}_{0,0}^+ - \mathcal{J}_{1,1}^+]. \quad (3)$$

If one adopts the free-current in the $q^+ = 0$ frame, the angular condition is satisfied at the first order in Q , but it is violated at the second order, for $Q^2 \rightarrow 0$ [1]. Therefore the angular condition is not a problem for the calculation of μ_d , while the quadrupole moment is not uniquely determined. Differently, following our model, both of them are well determined.

The matrix elements $\mathcal{J}_{S'_z, S_z}^\mu$ can be easily calculated, by using the action of the free current on a two-body state $|\vec{P}_\perp, P^+\rangle |\chi_{S, S_z}\rangle$ [16]:

$$\begin{aligned} \langle p'_1, p'_2; \sigma'_1, \sigma'_2 | J_{free}^\mu(0) | \vec{P}_\perp = 0, P^+ \rangle |\chi_{S, S_z}\rangle &= \sum_{\sigma_1''} \bar{w}(p'_1, \sigma'_1) [2m(f_e^{is}((p'_1 - p_1)^2) - \\ f_m^{is}((p'_1 - p_1)^2) \frac{(p_1 + p'_1)^\mu}{(p_1 + p'_1)^2} + f_m^{is}((p'_1 - p_1)^2) \gamma^\mu] w(p_1, \sigma_1'') \langle \vec{k}, \sigma_1'', \sigma'_2 | \chi_{S, S_z} \rangle \frac{1}{\xi} \end{aligned} \quad (4)$$

where $w(p, \sigma)$ is the front-form Dirac spinor [16], while f_e^{is} and f_m^{is} are the isoscalar electric and magnetic Sachs form factors of the nucleon. The relations between the internal (\vec{k}_\perp, k_z) and individual nucleon variables in our reference frame are given by

$$\vec{p}_{1\perp} = \vec{p}'_{1\perp} = \vec{k}_\perp, \quad p_1^+ = \xi P^+, \quad k_z = \omega(k)(2\xi - 1), \quad \xi' = 1 + (\xi - 1)P^+/P'^+ \quad (5)$$

where $\omega(k) = \sqrt{m^2 + k^2}$, with $m = (m_p + m_n)/2$ the nucleon mass, and $k = |\vec{k}|$. Nucleon form factors cannot be factorized in the current matrix elements, since from Eq. (5) one has $(p'_1 - p_1)^2 = -4Q^2(m^2 + \vec{k}_\perp^2)/(4m_d^2 \xi \xi')$.

In FFHD, the internal deuteron wave function with polarization vector \vec{e}_{S_z} is given by (cf. [7])

$$\langle \vec{k} | \chi_{1, S_z} \rangle = (2\pi)^{3/2} [\omega(k)/2]^{1/2} v(\vec{k})^{-1} v(-\vec{k})^{-1} [\varphi_0(k) \delta_{ij} - \frac{1}{\sqrt{2}} (\delta_{ij} - \frac{3k_i k_j}{k^2}) \varphi_2(k)] \sigma_i \sigma_j (e_{S_z})_j \quad (6)$$

where a sum over the repeated indices $i, j = 1, 2, 3$ is assumed, $v(\vec{k})$ is the generalized Melosh matrix [8] and σ_i are the Pauli matrices. The wave functions $\varphi_0(k)$ and $\varphi_2(k)$ coincide with the non-relativistic S and D waves in momentum representation [17] and are normalized so that $\int [\varphi_0(k)^2 + \varphi_2(k)^2] d^3\vec{k} = 1$.

Our FFHD results corresponding to different $N - N$ interactions are compared in Table 1 with the non-relativistic ones (for overcoming numerical instabilities a careful analytical reduction of Eqs. (2,3) is needed). The standard non-relativistic results obtained with a one-body current crucially depend on the asymptotic normalization ratio η of D and S wave functions and on the D -state percentage, P_D , but one cannot obtain at the same time the experimental values for both μ_d and Q_d . In our Poincaré covariant calculation the relativistic corrections (RC) bring both μ_d and Q_d closer to the experimental values, except for the charge-dependent Bonn interaction. In Ref. [7], RC have been calculated within FFHD by using the free current in the $q^+ = 0$ frame and they resulted to be very small for Q_d , while for μ_d were able to explain only part of the disagreement with the experimental value. It should be stressed that our current operator and the one used in Ref. [7] are different, since both of them are obtained from the free one, but in different reference frames, related by an interaction dependent rotation.

In Fig. 1, μ_d and Q_d are reported against the asymptotic normalization ratio, η . As already observed for the non-relativistic calculations of Q_d [20, 26], a remarkable linear behaviour appears for both quantities, except for the Bonn interaction. The values of μ_d and Q_d , suggested by this linear behaviour in correspondence of $\eta^{exp} = 0.0256$, differ from the experimental ones only by 0.5% and 2%, respectively, i.e. much less than for the non-relativistic results. The RC to μ_d are rather large and the total result is greater than μ_d^{exp} . This shows that, within our framework, even the sign of explicit contributions of two-body currents is different from the one needed in the non-relativistic case.

In summary, our results for μ_d and Q_d , unambiguously calculated by a Poincaré covariant current built up from the one-body current in the Breit reference frame where $\vec{P}_\perp = \vec{q}_\perp = 0$, show that the total contribution of explicit two-body currents (from meson-exchange, Z -graphs, etc.) and isobar configurations is relatively small at $Q^2 = 0$. It should be stressed that explicit two-body current contributions, considered in addition to the ones already contained in Eq.(1), must fulfill separately the constraints imposed by the extended Poincaré covariance, hermiticity and current conservation [8]. An evaluation in our Breit frame of explicit two-body contributions will be performed elsewhere.

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References

- [1] L.A. Kondratyuk and M.I. Strikman, Nucl. Phys. **A 426**, 575 (1984).
- [2] E. Hummel and J. A. Tjon, Phys. Rev. **C 42**, 423 (1990); ibidem **C 49**, 21 (1994).
- [3] J.W. Van Orden, N. Devine and F. Gross, Phys. Rev. Lett. **75**, 4369 (1995).
- [4] J. Carbonell, B. Desplanques, V.A. Karmanov and J.-F. Mathiot, Phys. Rep. **300**, 215 (1998).
- [5] L.P. Kaptari, A.Yu. Umnikov, S.G. Bondarenko, K.Yu. Kazakov, F.C. Khanna and B. Kämpfer, Phys. Rev. **C 54**, 986 (1996).

- [6] J. Carlson and R. Schiavilla, *Rev. Mod. Phys.* **70**, 743 (1998).
- [7] P.L. Chung, F. Coester, B.D. Keister and W.N. Polyzou, *Phys. Rev. C* **37**, 2000 (1988); P.L. Chung, B.D. Keister, F. Coester, *Phys. Rev. C* **39**, 1544 (1989).
- [8] F.M. Lev, E. Pace, G. Salmè, *Nucl. Phys. A* **641**, 229 (1998).
- [9] P.A.M. Dirac, *Rev. Mod. Phys.* **21**, 392 (1949).
- [10] M.V. Terent'ev, *Sov. J. Nucl. Phys.* **24**, 106 (1976).
- [11] I.L. Grach and L.A. Kondratyuk, *Yad. Fiz.* **39**, 316 (1984).
- [12] V.A. Karmanov and A.V. Smirnov, *Nucl. Phys. A* **546**, 691 (1992); *Nucl. Phys. A* **575**, 520 (1994).
- [13] E.L. Lomon, *Ann. Phys. (N.Y.)* **125**, 309 (1980).
- [14] R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, *Phys. Rev. C* **51**, 38 (1995).
- [15] E. Pace, G. Salmè, F. Lev, *Proceedings of the Workshop on "Electron Nucleus Scattering"*, ed. O. Benhar, A. Fabrocini and R. Schiavilla (Edizioni ETS, Pisa, 1999) p. 401 and to be published.
- [16] E. Pace, G. Salmè, F. Lev, *Phys. Rev. C* **58**, 2655 (1998).
- [17] F. Coester, S.C. Pieper and F.J.D. Serduke, *Phys. Rev. C* **11**, 1 (1975).
- [18] N.L. Rodning, L.D. Knutson, *Phys. Rev. C* **41**, 898 (1990).
- [19] I. Lindgren, in *"Alpha-, Beta-, and Gamma-Ray Spectroscopy"*, ed. K. Siegbahn (North-Holland, Amsterdam, 1965) Vol. 2, p. 1620.
- [20] T.E.O. Ericson and M. Rosa-Clot, *Nucl. Phys. A* **405**, 497 (1983).
- [21] R.V. Reid, *Ann. of Phys.* **50**, 411 (1968).
- [22] R.B. Wiringa, R.A. Smith and T.A. Ainsworth, *Phys. Rev. C* **29**, 1207 (1984).
- [23] M. Lacombe et al., *Phys. Rev. C* **21**, 861 (1980).
- [24] V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, J.J. de Swart, *Phys. Rev. C* **49**, 2950 (1994).
- [25] R. Machleidt, F. Sammarruca, Y. Song, *Phys. Rev. C* **53**, R1483 (1996).
- [26] S. Klarsfeld, J. Martorell and D.W.L. Sprung, *J. Phys. G: Nucl. Phys.* **10**, L205 (1984).

Table 1: Magnetic moment (in nuclear magnetons) and quadrupole moment for the deuteron, corresponding to different $N - N$ interactions; μ_d^{NR} and Q_d^{NR} are the nonrelativistic results, μ_d (LPS) and Q_d (LPS) our present results; P_D is the D -state percentage, and $\eta = A_D/A_S$ the asymptotic normalization ratio.

Interaction	P_D	η	μ_d^{NR}	μ_d (LPS)	$Q_d^{NR} (fm^2)$	Q_d (LPS) (fm^2)
Exp		0.0256(4) [18]		0.857406(1) [19]		0.2859(3) [20]
RSC [21]	6.47	0.0262	0.8429	0.8611	0.2796	0.2852
Av14 [22]	6.08	0.0265	0.8451	0.8608	0.2860	0.2907
Paris [23]	5.77	0.0261	0.8469	0.8632	0.2793	0.2841
Av18 [14]	5.76	0.0250	0.8470	0.8635	0.2696	0.2744
Nijm93 [24]	5.75	0.0252	0.8470	0.8629	0.2706	0.2750
RSC93 [24]	5.70	0.0251	0.8473	0.8637	0.2703	0.2750
Nijm1 [24]	5.66	0.0253	0.8475	0.8622	0.2719	0.2758
CD-Bonn [25]	4.83	0.0255	0.8523	0.8670	0.2696	0.2729

Figure Caption

Fig. 1. (a) Deuteron magnetic moment, μ_d , as a function of the asymptotic normalization ratio η , for different $N - N$ interactions. The full dot represents the experimental values for μ_d and η ; empty triangles and diamonds correspond to the non-relativistic and relativistic results of Table 1, respectively, while the solid and dashed lines are linear fits for these results. Full triangle and diamond are the results of the CD-Bonn interaction. (b) The same as in (a), but for the deuteron quadrupole moment, Q_d .

